

# A Multicomposite, Multilayered Cylindrical Dielectric Resonator for Application in MMIC's

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## Abstract

A brief summary is given of a new rigorous method to determine resonant frequencies and field distributions of all resonant modes in a multicomposite, multilayered cylindrical dielectric resonator. This resonator consists of a number of concentrical cylinders, which are arbitrarily layered in axial direction. As examples, a dielectric sphere and a dielectric cone placed in MIC environment have been analysed. The sphere and the cone are structurally approximated by bodies of revolution with a stepped cross section. The calculated resonant frequencies have an accuracy of  $<10^{-4}$ .

## I. Introduction

With increasingly wide applications of dielectric resonators in monolithic microwave integrated circuits (MMIC's), especially at high frequency, new dielectric resonator structures are demanded to design. The new resonators should have special resonant field distributions, so that they can be easily coupled to the small size MMIC's [1]. These requirements in total only can be fullfilled, if unconditional dielectric resonator structures are considered for these applications. From this, a new rigorous analysis method, based on mode matching technique, to determine resonant frequencies and field distributions of TE modes in a very generalized cylindrical dielectric resonator was introduced in a previous work [1]. This resonator consists of a number of concentric cylinders, which are arbitrarily layered in axial direction, as shown in Fig. 1. It was called a multicomposite, multilayered cylindrical dielectric resonator. In this paper, the theory is extended to study not only TE modes, but also TM and hybrid modes in the resonator (Fig. 1).

As an example, a dielectric sphere placed on a substrate and between two parallel conducting plates has been analysed. The sphere is structurally approximated by a circumscribed and an inscribed dielectric body of revolution with a stepped cross section, respectively

(Fig. 2a and 2b). Thus the spherical dielectric resonator can be treated as a special case of the multicomposite, multilayered cylindrical dielectric resonator. If sufficiently many steps ( $N$ ) are selected, the solutions of the both resonators (Fig. 2a and 2b) should converge to the solution of the spherical dielectric resonator. The spherical dielectric resonator is very useful for application at millimeter-wave frequency, because the spherical sample is easier to produce in small dimension than the cylindrical ones. Different spherical dielectric resonators have been discussed mostly in spherical coordinate system [2-4]. The resonant frequency of the  $TE_{011}$  mode for a dielectric sphere placed between two parallel conducting plates was calculated by Vincent [5]. Comparisons with his numerical results have been made, showing a very good agreement.

As second example, a dielectric cone in the same MIC environment (Fig. 3) has been analysed. The conical dielectric resonator has the advantage that the magnetic field of the  $TE_{011}$  mode near the surface of the substrate is concentrated in the vicinity of the cone apex. Hence, it is possible to couple the resonator directly to a microstrip line or to a coplanar waveguide on a MMIC chip, not interfering with other circuits or components on it. Moreover, the undesired higher order TE modes are effectively suppressed.

## II. Theory

A multicomposite multilayered cylindrical dielectric resonator can be considered to be cascaded from many parallel-plate radial waveguides, which have the same distance between two parallel conducting plates and are layered by different dielectric materials in axial direction (Fig. 1). The fields in each radial waveguide can be presented in terms of a series of the TE and TM waveguide modes, whose unknown coefficients are the mode amplitudes at two boundary surfaces  $\rho=r_{j-1}$  and  $r_j$ . The boundary mode amplitudes of each two neighbouring waveguides are connected by a "cascade matrix"

derived from mode matching technique. The resonant frequencies of the resonator are determined by solving an eigenvalue problem of a linear homogeneous system of order  $N^{TE}+N^{TM}$  ( term numbers of the field series of the TE and TM modes ). This method can further be extended to study a multilayered, multicoated cylindrical dielectric resonator, using axial mode matching technique.

### III. Numerical Results

A dielectric sphere and a dielectric cone in a MIC environment ( Fig. 2 and 3 ) have been theoretically investigated. Fig. 4 shows the convergence of the resonant frequency for the  $TE_{011}$  mode when two bodies of revolution with a stepped cross section converge structurally to the sphere by increasing the step number N ( Fig. 2 ). If the step number N is selected larger than 50 and the term number of the field series  $N^{TE}$  larger than 20, the relative error of the resonant frequency for  $TE_{011}$  mode is smaller than  $10^{-4}$ . Fig. 5 shows the resonant frequencies of two non-leaky modes (  $TE_{011}$  and  $TE_{012}$  modes ) versus the sphere diameter. Fig. 6 shows the field pattern of the two modes for a dielectric sphere with diameter  $D=1.85$  mm. Fig. 7 shows the field pattern of three lowest TM modes for the dielectric sphere placed in a cylindrical conducting cavity with  $R=2.60$  mm and  $H=3.00$  mm. The resonant frequencies of these modes decrease with increasing the cavity radius R ( Fig. 8 ). Comparisons with the results by P. Vincent [5] for a dielectric sphere placed between two parallel conducting plates are shown in Fig. 9 and 10. They show a very good agreement.

Fig. 11 shows resonant frequencies of the single non-leaky mode (  $TE_{011}$  mode ) in the conical dielectric resonator ( Fig. 3 ) with different cone diameters D and heights L. Fig. 12 shows its field pattern with cone diameter  $D=2.55$  mm and height  $L=1.8$  mm. It is quite evident that the magnetic field strength near the substrate surface diminishes rapidly along radial direction away from the cone apex. Hence, a proper designed cone resonator can be coupled to a circular microstrip line or to a circular coplanar waveguide on a MMIC chip, not interfering with other circuit elements.

### IV. Conclusions

Computer programs have been developed in C language to study a multicomposite, multilayered cylindrical dielectric resonator and to draw the field pattern of the various resonant modes. A dielectric sphere and a dielectric cone placed in a MIC environment have been theoretically investigated. A proper designed conical dielectric resonator may be coupled to the small size MMIC's, using the magnetic field near the cone apex.

### References

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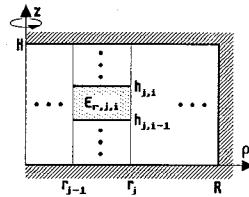


Fig. 1: A multicomposite multilayered cylindrical dielectric resonator structure.

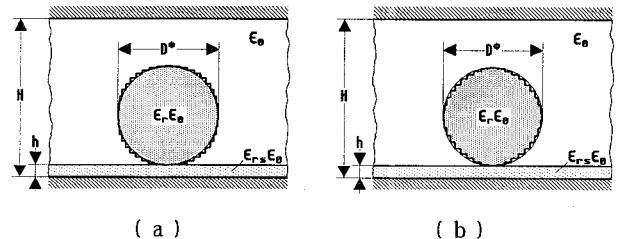


Fig. 2: A dielectric sphere, structurally approximated (a) by a circumscribed and (b) by an inscribed body of revolution with a stepped cross section ( step number  $N=10$  ), respectively, placed on a substrate and between two parallel conducting plates.

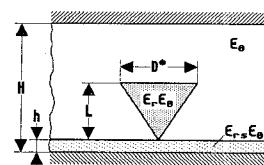


Fig. 3: A conical dielectric resonator structure.

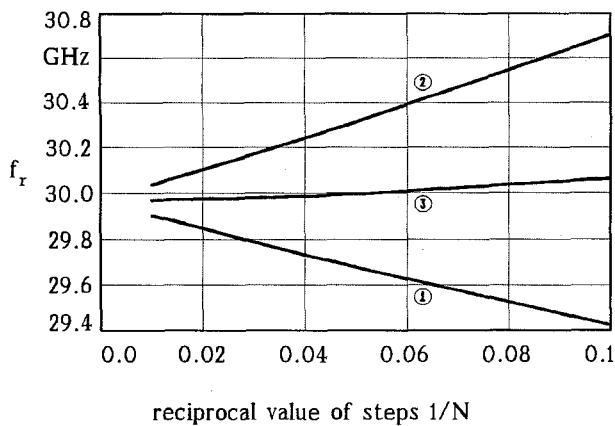


Fig. 4: Resonant frequency of the  $TE_{011}$  mode vs steps N, ( 1 ) for a circumscribed and ( 2 ) for an inscribed body of revolution with a stepped cross section in the MIC environment ( Fig. 2 ) with  $D=1.85$  mm,  $\epsilon_r=29.57$ ,  $h=250$   $\mu$ m,  $\epsilon_{rs}=10$ ,  $H=3.00$  mm and  $N^{TE}=20$  is selected. ( 3 ) average value of the resonant frequencies.

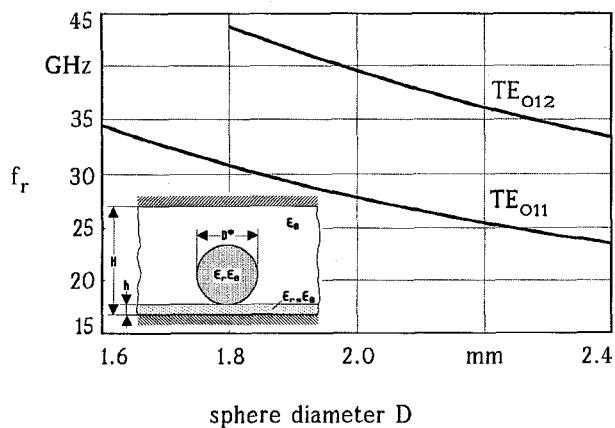


Fig. 5: Resonant frequency of the  $TE_{011}$  and  $TE_{012}$  modes vs sphere diameter, for a spherical dielectric resonator with  $\epsilon_r=29.57$ ,  $h=250$   $\mu$ m,  $\epsilon_{rs}=10$ ,  $H=3.00$  mm.

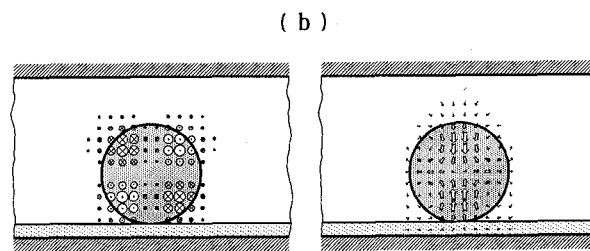


Fig. 6: Electric ( left ) and magnetic ( right ) field pattern of ( a )  $TE_{011}$  and ( b )  $TE_{012}$  modes in a spherical dielectric resonator with  $D=1.85$  mm,  $\epsilon_r=29.57$ ,  $h=250$   $\mu$ m,  $\epsilon_{rs}=10$ ,  $H=3$  mm.

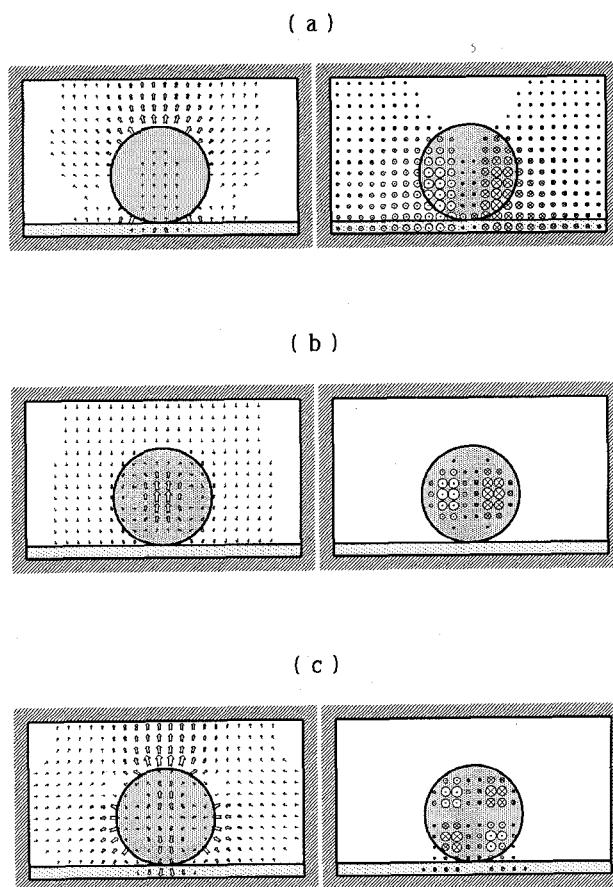


Fig. 7: Electric ( left ) and magnetic ( right ) field pattern of ( a )  $TM_{H,010}$ , ( b )  $TM_{D,011}$  and ( c )  $TM_{D,012}$  modes, for the spherical dielectric resonator ( Fig. 6 ) in a cylindrical conducting cavity with  $H=3$  mm,  $R=2.6$  mm.

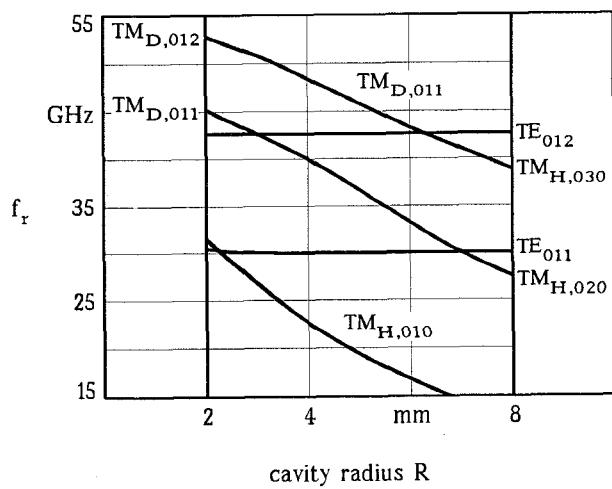


Fig. 8: Resonant frequency variations of some lower order modes vs cavity radius  $R$ , for the spherical dielectric resonator (Fig. 7).

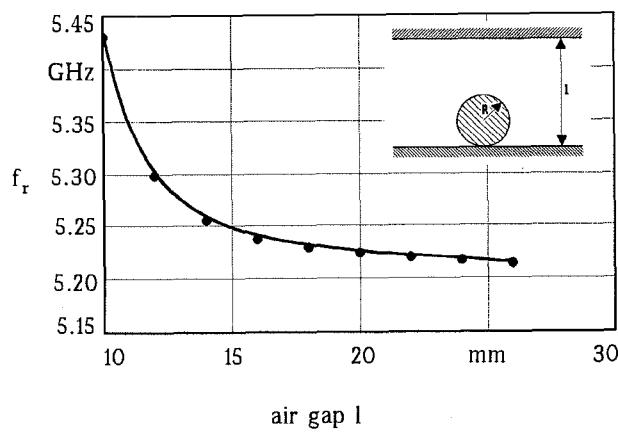


Fig. 9: Resonant frequency of the  $TE_{011}$  mode vs air gap between the two conducting plates, for a dielectric sphere ( radius  $R=5$  mm, permittivity  $\epsilon_r=34.61$  ); —: our computation with  $N^{TE}=20$  terms of the field series and  $N=50$  steps in structure, • • •: numerical results by P. Vincent [5].

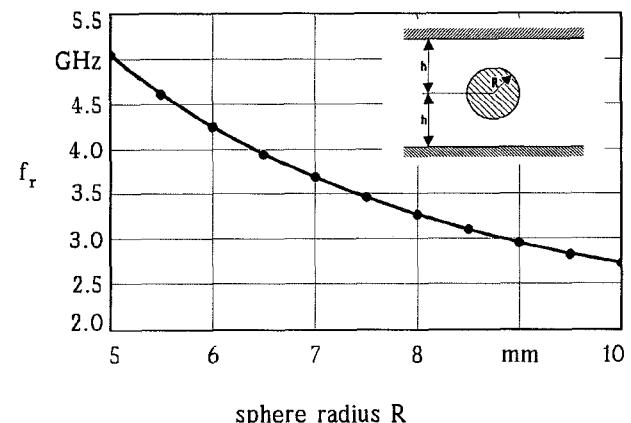


Fig. 10: Resonant frequency of the  $TE_{011}$  mode vs sphere radius  $R$  with permittivity  $\epsilon_r=34.61$ , air gap  $2h=20$  mm. —: our computation with  $N^{TE}=20$  terms of the field series and  $N=50$  steps in structure, • • •: numerical results by P. Vincent [5].

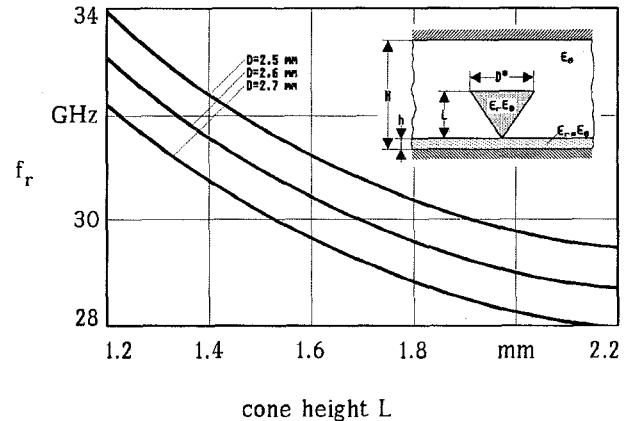


Fig. 11: Resonant frequency of the  $TE_{011}$  mode vs cone height  $L$ , for a conical dielectric resonator with different cone diameters  $D$ ,  $\epsilon_r=29.57$ ,  $h=250$   $\mu$ m,  $\epsilon_{rs}=10$ ,  $H=3.0$  mm.

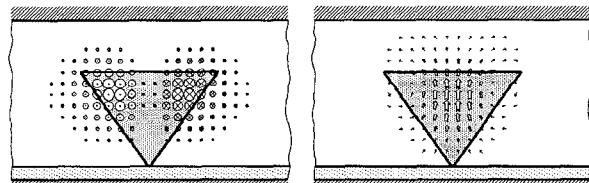


Fig. 12: Electric ( left ) and magnetic ( right ) field pattern of the  $TE_{011}$  mode in a conical dielectric resonator ( Fig. 3 ) with  $D=2.55$  mm,  $L=1.80$  mm,  $\epsilon_r=29.57$ ,  $h=250$   $\mu$ m,  $\epsilon_{rs}=10$ ,  $H=3$  mm.